

# Chapter 6: Polygons

## Section 4-5: Rhombuses, rectangles and squares

### Key Learning Objective: special parallelograms

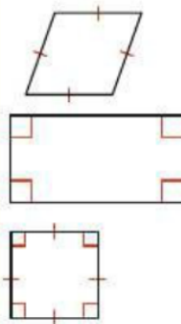
#### Definition

A **rhombus** is a parallelogram with four congruent sides.

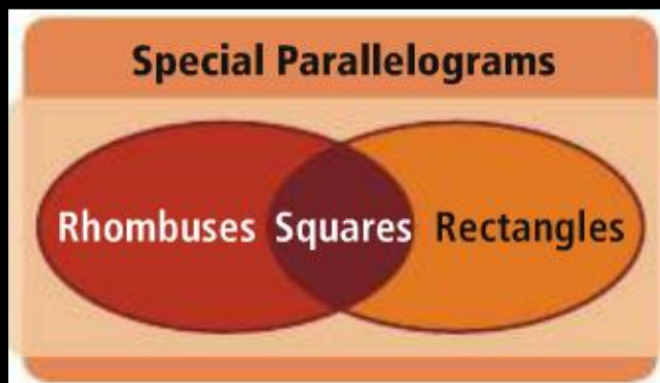
A **rectangle** is a parallelogram with four right angles.

A **square** is a parallelogram with four congruent sides and four right angles.

#### Diagram



## Visualization: venn diagram



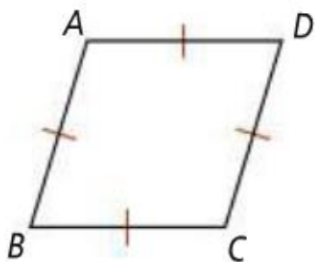
## Real-life example: rectangle?

- ▶ Is parallelogram ABCD a rhombus, a rectangle or a square?
- ▶ Apply the properties of a parallelogram: opposite angles are congruent, so angle D must be 90 degrees.
- ▶ Check the properties of a rhombus, rectangle and square. Which one matches that information?
- ▶ Rectangle!
- ▶ What is parallelogram EFGH?

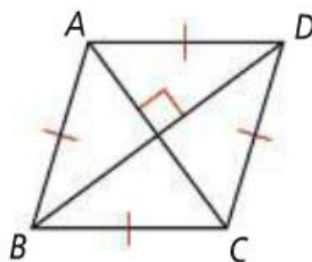


## Theorem 6-13

If ...

 $ABCD$  is a rhombus

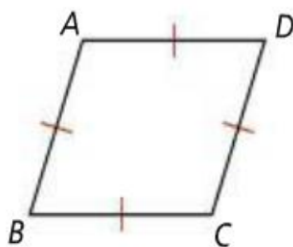
Then ...

 $\overline{AC} \perp \overline{BD}$ 

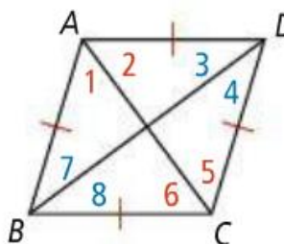
- ▶ If a parallelogram is a rhombus, then its diagonals are perpendicular.

## Theorem 6-14

If ...

 $ABCD$  is a rhombus

Then ...



$\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$

$\angle 5 \cong \angle 6$

$\angle 7 \cong \angle 8$

- ▶ If a parallelogram is a rhombus then each diagonal bisects a pair of opposite angles.

## Finding missing angle measures

What are the measures of the numbered angles in rhombus  $ABCD$ ?

$m\angle 1 = 90$  The diagonals of a rhombus are  $\perp$ .

$m\angle 2 = 58$  Alternate Interior Angles Theorem

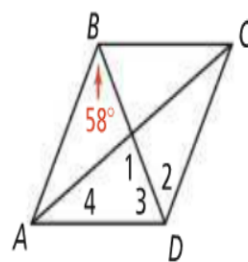
$m\angle 3 = 58$  Each diagonal of a rhombus bisects a pair of opposite angles.

$m\angle 1 + m\angle 3 + m\angle 4 = 180$  Triangle Angle-Sum Theorem

$90 + 58 + m\angle 4 = 180$  Substitute.

$148 + m\angle 4 = 180$  Simplify.

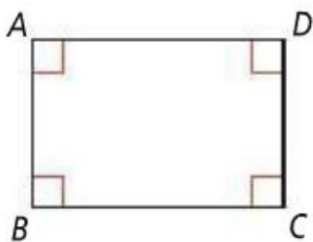
$m\angle 4 = 32$  Subtract 148 from each side.



## Theorem 6-15

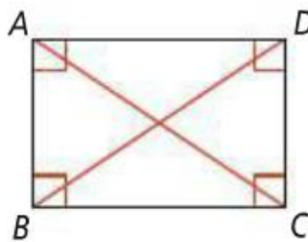
If ...

$ABCD$  is a rectangle



Then ...

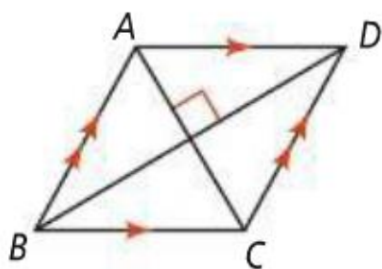
$\overline{AC} \cong \overline{BD}$



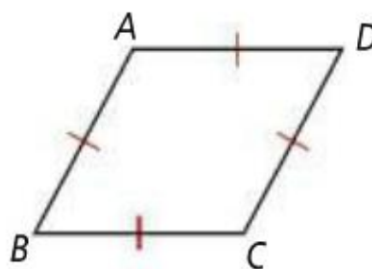
- If a parallelogram is a rectangle then its diagonals are congruent.

## Theorem 6-16

If ...

 $ABCD$  is a  $\square$  and  $\overline{AC} \perp \overline{BD}$ 

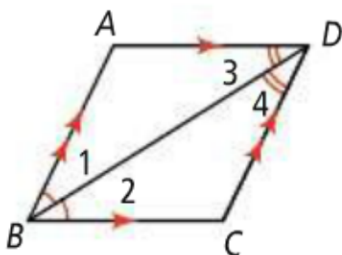
Then ...

 $ABCD$  is a rhombus

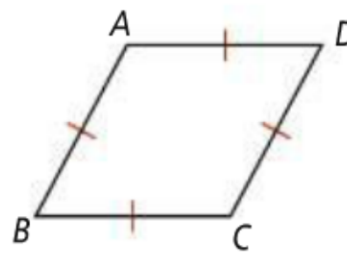
- ▶ If the diagonals of a parallelogram are perpendicular then the parallelogram is a rhombus.

## Theorem 6-17

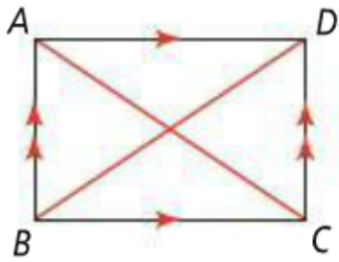
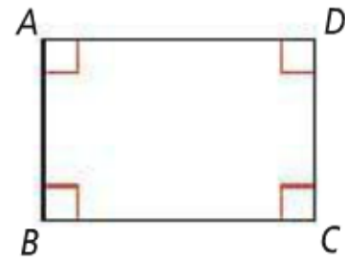
If ...

 $ABCD$  is a  $\square$ ,  $\angle 1 \cong \angle 2$ , and  $\angle 3 \cong \angle 4$ 

Then ...

 $ABCD$  is a rhombus

- ▶ If one diagonal of a parallelogram bisects a pair of opposite angles then the parallelogram is a rhombus.

**Theorem 6-18****If . . .** $ABCD$  is a  $\square$ , and  $\overline{AC} \cong \overline{BD}$ **Then . . .** $ABCD$  is a rectangle

- If the diagonals of a parallelogram are congruent then the parallelogram is a rectangle.